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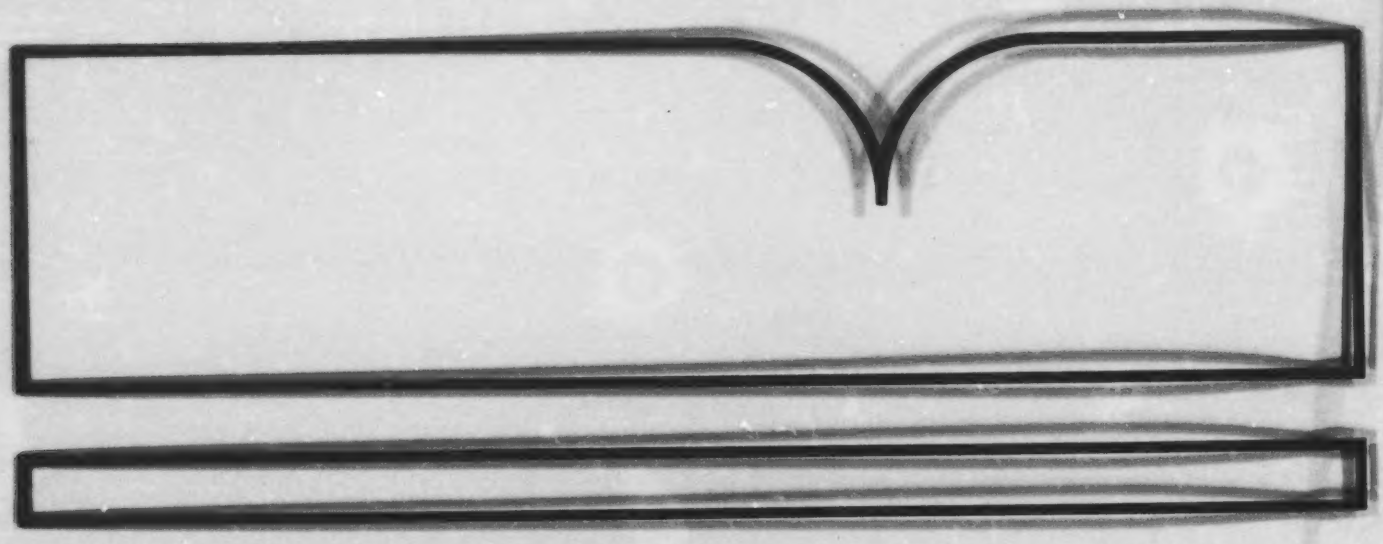

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JOURNAL ARTICLE

AN APPLICATION OF GAME THEORY IN THE DESIGN OF OPTIMAL AIR POLLUTION CONTROL MEASURES

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Abstract—Methods of game theory are used to develop a technique for finding optimal solutions to air quality problems that involve multiple objectives. The technique is demonstrated using a hypothetical problem in which sites for two new power plants are sought that optimally satisfy five objectives subject to given constraints. The five objectives are minimization of (1) fuel and pollutant control costs, (2) long-term averaged SO_2 concentration, (3) peak short-term SO_2 concentration, (4) long-term averaged particulate concentration, and (5) peak short-term particulate concentration. The constraints include the number and locations of fuel sources, the energy and pollutant contents and cost of fuel from each source, the admissible areas for plant locations, etc. The focus of the paper is on concepts and technique development rather than the solution of specific problems.

1. INTRODUCTION

Since air pollution models are used extensively to assess the effectiveness of proposed solutions to air pollution problems, it would be advantageous to have a means by which a computer could itself use models to find optimal solutions to given problems. Land use planning, the design of emissions control systems for power plants and the formulation of secondary air quality standards are three examples of areas in which one faces problems with given constraints and solutions are sought that optimally satisfy a number of objectives simultaneously. Dealing with problems of this kind has not been practical heretofore because techniques have not been available for applying air pollution models to multi-criteria optimization problems. This study uses concepts of game theory to develop a technique of this kind.

We shall work within the context of a specific problem that contains elements of each of the three problem areas cited above, namely the selection of sites for new power plants that achieve optimal trade-offs between capital and operating costs on the one hand and the degradation of air quality on the other hand. The measures of air quality considered are a long-term and a short-term averaged concentration of each of two pollutants, a total of four measures in all. The costs taken into consideration include fuel cost, the cost of transporting fuel from given sources (such as coal mines) to the given plant site and the cost of emissions controls (such as flue gas desulfurization and fly ash precipitation). Optimal trade-offs among the air quality and cost objectives are sought within the space of feasible solutions using concepts of game theory. Here a 'feasible' solution is a plant site, fuel allocation and emissions control design that jointly satisfy all constraints imposed on the system. In the case of power plant siting, constraints include the number and geographical locations of fuel sources, the energy and pollutant contents and the cost of fuel from each source, the required generating capacity of the plant, the geographical areas in which plant construction is admissible, etc.

The basic aspects of the optimal siting problem can be illustrated with the aid of Fig. 1. Shown there is a hypothetical situation in which a coal-fired power plant of given generating capacity r is to be located in a region where polluting industries already exist. The objective is to find a site within the admissible region, indicated by the shaded areas in Fig. 1, at which the cost of holding emissions to a level consistent with the allowable air pollution increment is minimum. For a given plant site, $\mathbf{x}$, the total operating costs consist in part of the sum of the costs of the coal purchased from each mine, and the cost of transporting the coal from each mine to the plant site $\mathbf{x}$. The amount of coal taken from each mine must in turn be compatible with both the supply capacities of each mine and the plant's generating capacity r, taking into account differences in the energy content of the coal from each mine. The fuel allocation in turn determines the degree of emissions control that the plant must maintain to prevent the effluent concentrations from exceeding the air pollutant increment for that area. We assume that the fly ash and sulfur content of the coal from each mine is known.

Within this framework the need for emissions control can be handled by defining sources of virtual coal, each of whose ash, sulfur and energy contents are *negative* and whose cost is given per unit of negative ash or sulfur. The negative energy contents of the

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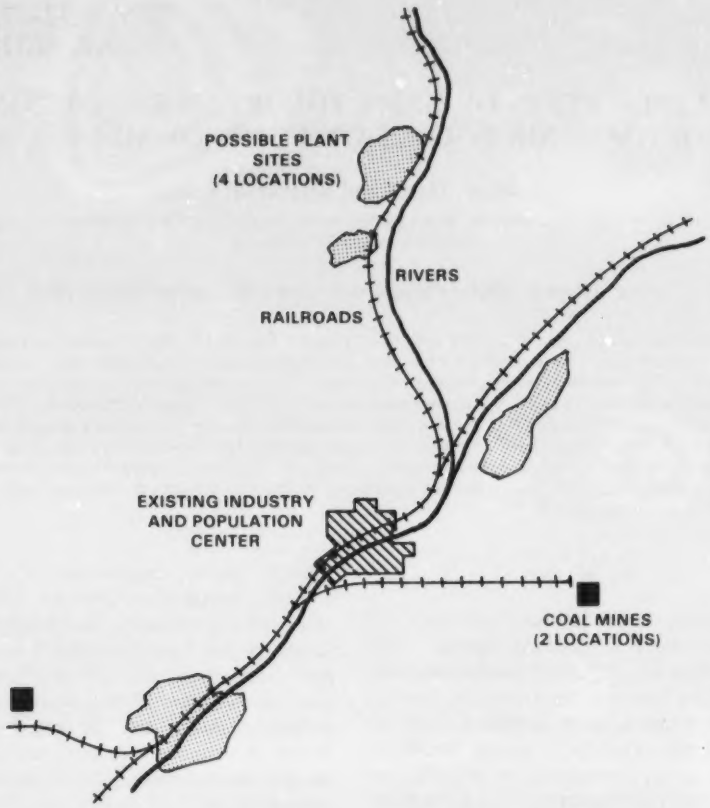

Fig. 1. Hypothetical example of possible sites (shaded areas) for new power plants and their relationship to existing industry locations and fuel sources.

virtual coal reflect the fact that sulfur scrubbers and fly ash precipitation consume a portion of the power plants output energy. In the optimization algorithm, the energy requirements of the power plants are met by drawing upon the reserves of real coal and the air quality requirements are satisfied by the addition of virtual coal of either the negative sulfur or negative ash types, or both.

To elaborate briefly on the virtual coal concept, suppose coal is available from two mines with percentage sulfur contents s_i $(i = 1, 2)$, energy contents k_i (calories/mass) and costs c_i (\$/mass). If a given plant consumes coal from each mine at a rate χ_i (mass/time), the uncontrolled emission rate (mass/time) of sulfur would be

$$Q = \chi_1 s_1 + \chi_2 s_2$$

and the corresponding energy output and fuel cost would be

$$E = \chi_1 k_1 + \chi_2 k_2$$

and

$$C = \chi_1 c_1 + \chi_2 c_2,$$

respectively. Using the virtual coal concept we can express the sulfur emission rate Q_c of a controlled source in the form

$$Q_c = \chi_1 s_1 + \chi_2 s_2 - \chi_v s_v \tag{1}$$

where χ_v is the effective consumption rate (mass/time) of virtual coal whose sulfur content is $-s_v$ (%). Using the conventional method we have

$$Q_c = (1 - \lambda)(\chi_1 s_1 + \chi_2 s_2) \tag{2}$$

where λ is the efficiency of the scrubber $(0 \leqslant \lambda \leqslant 1)$. Combining (1) and (2) we see that the action of a scrubber of efficiency λ is equivalent to the consumption of virtual coal at a rate

$$\chi_v = \lambda(\chi_1 s_1 + \chi_2 s_2) s_v^{-1} \tag{3a}$$

from a source whose sulfur content is (arbitrarily)

$$s_v = -(s_1 + s_2)/2. \tag{3b}$$

If a scrubber operating at efficiency λ consumes energy at a rate $k_v(\lambda)$ (calories/mass of coal combustion products processed), the energy output of the controlled plant would be

$$E = \chi_1 k_1 + \chi_2 k_2 - (\chi_1 + \chi_2) k_v(\lambda) \tag{4}$$

and the effective fuel cost would be

$$C = \chi_1 c_1 + \chi_2 c_2 + (\chi_1 + \chi_2) c_v(\lambda), \quad (5)$$

where $c_v(\lambda)$ is the operating cost (dollars/mass of coal burned) of a scrubber of efficiency λ. Since $k_v(\lambda)$ and $c_v(\lambda)$ are in principle known functions, both approaching zero as $\lambda \to 0$ and increasing, perhaps exponentially as $\lambda \to 1$, the plant's output energy and effective fuel cost is set once we have selected a value for λ that allows us to achieve a given sulfur emissions reduction.

For a given allocation of fuel from the mines to the plant the uncontrolled emissions of the plant can be estimated; and with this knowledge and suitable atmospheric transport models, the impact of the plant on air quality can be determined. Comparing the predicted pollutant concentrations with the allowed pollutant increments determines the degree of emissions control that must be imposed, and this in turn establishes the total emissions control necessary and hence the operating cost of the plant at the given site. Thus, the problem of optimal plant siting requires an iterative manipulation of the design variables, namely fuel allocation, emissions control and plant location, until the configuration of least cost is found. Two essential elements are required to implement this process: (1) an atmospheric transport model that can predict the air quality impact of given plant emissions, and (2) a rational procedure for manipulating the design variables. The latter is a critical component of any optimization scheme because without an efficient means of scanning the phase space of the design variables, the solutions produced may be artifacts of the scheme, rather than optimal points; and/or the computer time required to generate them may be prohibitive. The method we use in the present study is a mathematically sophisticated procedure that is based on the concepts of game theory. It yields solutions in which the design objectives are optimized jointly. That is, each objective is satisfied to the fullest extent possible without unduly restricting the degree to which each of the other objectives can be met.

In this regard it is important to realize that under current U.S. air pollution regulations, a new industry that is to be located in a region where air quality standards are in attainment is forbidden to emit pollutants at a rate that would cause concentrations to exceed the standard. In this case the pollutant concentrations that the industry would produce become a part of the constraints on the system design rather than a variable that is to be minimized. As far as our optimization procedure is concerned, treating air quality impact as a constraint is a trivial task. Therefore, to provide a more interesting demonstration of our technique, we will consider problems in which the air quality impact is a variable to be minimized jointly with cost, rather than a variable that is limited by constraints.

This study is not the first attempt to apply optimization techniques to problems of air pollution control. Seinfeld and his co-workers have reported a number of efforts along this line. Seinfeld and Kyan (1971) considered the problem of finding the emissions control strategy for sources in a given airshed that would yield a given level of air quality at least cost. This work was extended by Kyan and Seinfeld (1972) to treat the case of control plans that are implemented over a period of years. Again, the objective was to find the least cost (multiyear) emissions control strategy that would maintain a given level of air quality, taking into account that the cost of controlling a given source in a given year is a function of the type of control equipment that was installed in previous years.

The major difference between the present study and the earlier studies with which we are familiar is our consideration of multiple objectives in the optimization process. That is, instead of considering the cost of emissions controls as the only variable to be minimized, subject to constraints imposed by air quality standards, we consider the control costs and pollutant concentrations as a set of variables that are to be minimized jointly. In the field of decision theory a number of techniques have been developed for treating multiple objective problems (see, for example, Zeleny, 1982). In this paper we will utilize a method based on the concepts of game theory first developed by Von Neumann and advanced by Von Neumann and Morgenstern (1947). One of the advantages of our approach is that it reveals the bounds on each of the objective variables that are achievable when any one of the variables is minimized alone. The information provided by these bounding values is potentially very valuable in determining where the limits of feasible air quality standards lie.

Following a brief discussion in the next section of the basic equations for short- and long-term averaged concentration, we develop the optimization technique itself in section 3. An elementary application of the overall method is illustrated in section 4.

2. MATHEMATICAL RELATIONSHIP BETWEEN POLLUTANT EMISSIONS AND CONCENTRATIONS

The transport model provides the mathematical link between the given emissions of a source and its resulting impact on air quality. The National Ambient Air Quality Standards which have been established in the U.S. for some pollutants, e.g. SO_2, are expressed in terms both of an annual mean concentration and a short-term (24-h) average concentration that is not to be exceeded more than once a year. These are the two measures of environmental impact that we will consider in this report to illustrate our proposed methods of model applications.

Lamb (1976), referred to hereafter as L76, developed mathematical expressions for both the long-term mean concentration and its higher order moments, from which estimates of the peak short-term averaged concentration can be determined. His expression for the annual averaged concentration at a receptor

located at point $\mathbf{r}$ at time-of-day ξ, where $0 \leqslant \xi < 24$ h, is

$$\langle c(\mathbf{r}, \xi) \rangle = \iint_{-\infty}^{\xi} p(\mathbf{r}, \xi | \mathbf{r}', t') S(\mathbf{r}', t') \mathrm{d}t' \, \mathrm{d}\mathbf{r}' \quad (6)$$

where

$S(\mathbf{r}', t')$ = source strength at point $\mathbf{r}'$ at time-of-day t', and it is assumed that S has diurnal periodicity, i.e.

$$S(\mathbf{r}', t') = S(\mathbf{r}', t' + 24 \text{ h}).$$

Also, in (6) the kernel $p(\mathbf{r}, \xi | \mathbf{r}', t')\mathrm{d}\mathbf{r}$ is the frequency with which particles released at $\mathbf{r}'$ are found in the small volume $\mathrm{d}\mathbf{r}$ centered at $\mathbf{r}$ at time ξ after a travel time of $(\xi - t')$. In the case of inert species or those that undergo linear chemical reactions, the kernel p is a Lagrangian, climatological property of the atmosphere in the region of the source and receptor points $\mathbf{r}'$ and $\mathbf{r}$.

It is also shown in L76 that the probability that the concentration at a given receptor $\mathbf{r}$ will exceed a given value c_m, say, for a given length of time, say τ, beginning at time-of-day ξ is given by

$$\text{prob}[c(\mathbf{r}, t) > c_m \text{ for } \xi \leqslant t \leqslant \xi + \tau] \leqslant \frac{R(\tau; \mathbf{r}, \xi)}{c_m^2} \quad (7)$$

where

$$R(\tau; \mathbf{r}, \xi) = \iiint_{-\infty}^{\xi} \int_{-\infty}^{\xi+\tau} p_2(\mathbf{r}, \xi; \mathbf{r}, \xi + \tau | \mathbf{r}', t', \mathbf{r}'', t'') \, S(\mathbf{r}', t') \times S(\mathbf{r}'', t'') \mathrm{d}t' \, \mathrm{d}t'' \, \mathrm{d}\mathbf{r}' \, \mathrm{d}\mathbf{r}'' \quad (8)$$

where the kernel p_2 is another (known) climatological property of the given region. If the air quality standard stipulated that the 24-h averaged concentration could not exceed a value Γ more than once a year, then by casting the 'once a year' condition into a probability equivalent and setting $\tau = 24$ h and $c_m = \Gamma$, we would have in (7) and (8) a quantitative condition that our source distribution S would have to meet to comply with the standards. Thus, Equations (6)–(8) provide a link between the pollutant emissions and the resulting air quality, given the climatological properties of the region in the Lagrangian forms p and p_2.

Since the principal focus of this paper is on optimization techniques, we would like to keep the transport models as simple as possible. To achieve this we will restrict attention to problems involving steady, point sources only. With this restriction (8) can be expressed as

$$R(\tau; \mathbf{r}, \xi) = P_2(\mathbf{r}, \tau, \xi; \mathbf{X}_N, \mathbf{Q}_N) \quad (9)$$

where $\mathbf{X}_N = (\mathbf{r}_{S1}, \mathbf{r}_{S2}, \ldots \mathbf{r}_{SN})$ is a vector representing the locations of N point sources whose strengths are represented by $\mathbf{Q}_N = Q_1, Q_2, \ldots Q_N$; and

$$P_2(\mathbf{r}, \tau, \xi; \mathbf{X}_N, \mathbf{Q}_N) = \sum_{i=1}^{N} \sum_{j=1}^{N} Q_i Q_j \int_{-\infty}^{\xi} \int_{-\infty}^{\xi+\tau} p_2 (\mathbf{r}, \xi; \mathbf{r}, \xi + \tau | \mathbf{r}_{Si}, t', \mathbf{r}_{Sj}, t'') \mathrm{d}t' \, \mathrm{d}t''. \quad (10)$$

Referring to (7), let us assume that the probability that the concentration at a given receptor site $\mathbf{r}$ will exceed a value C_M for a period τ beginning at hour ξ_i, $i = 1, \ldots 24$, of any day of the year is

$$\text{prob}[c(\mathbf{r}, t) > C_M \text{ for } \xi_i \leqslant t \leqslant \xi_i + \tau] = R(\tau; \mathbf{r}, \xi_i) C_M^{-2} \quad (11)$$

Combining (9) and (11) and assuming that the probabilities of exceedances at hours ξ_i and ξ_j, $i \neq j$, are approximately independent, we arrive at an expression for the probability ρ of at least one exceedance of the value C_M for a period τ any time in the year at site $\mathbf{r}$, namely

$$\rho(\mathbf{r}, \tau, C_M; \mathbf{X}_N, \mathbf{Q}_N) \simeq 1 - \prod_{i=1}^{24} [1 - C_M^{-2} P_2 (\mathbf{r}, \tau, \xi_i; \mathbf{X}_N, \mathbf{Q}_N)]. \quad (12)$$

We could use this expression, for example, to estimate the probability that the N sources will cause a violation of the 24-h SO_2 standard by setting $\tau = 24$ h and $C_M = 365\ \mu\text{g m}^{-3}$. One of the objectives of our source siting optimization procedure will be to minimize the quantity

$$C_S(\mathbf{X}_N, \mathbf{Q}_N) = \max_{\mathbf{r} \in D} [\rho(\mathbf{r}, \tau, C_M; \mathbf{X}_N, \mathbf{Q}_N)] \quad (13)$$

which is the highest probability of an exceedance of a given concentration C_M for a given period τ anywhere in the airshed D. A second objective will be to minimize the annual average concentration caused by the sources, namely (see 6)

$$C_L(\mathbf{X}_N, \mathbf{Q}_N) = \max_{\mathbf{r} \in D} \left[\frac{1}{24} \sum_{i=1}^{24} \sum_{n=1}^{N} Q_n \int_{-\infty}^{\xi_i} \times p(\mathbf{r}, \xi_i | \mathbf{r}_{Sn}, t') \mathrm{d}t' \right]. \quad (14)$$

In summary, Equations (13) and (14) express the effect of emissions from N point sources of given locations, each with a given strength Q_n, on the short- and long-term averaged air quality, respectively, in a given airshed. The kernels P_2 and p that enter in the expressions are Lagrangian, climatological properties of the airshed which we assume are known.

3. OPTIMAL SOLUTIONS TO THE PLANT SITING PROBLEM

Let us first define the variables that we will use in formulating the problem mathematically:

$$\mathbf{z} = \text{vector of design variables} = \begin{bmatrix} \mathbf{z}_L \\ \mathbf{z}_F \end{bmatrix} \quad (15)$$

where

$\mathbf{z}_F = (\chi_{11}, \chi_{12}, \ldots \chi_{MN})$ = vector of fuel allocation parameters

$\mathbf{z}_L = (\mathbf{X}_N)$ = vector of coordinates of the power plant sites;
M = number of sources of fuel, such as coal mines, including virtual sources;
N = number of power plants to be located;
χ_{ij} = flow rate of fuel from mine i to plant j (see (3a) for flow rate of virtual coal);
d_{ij} = upper bound restriction on the flow rate χ_{ij};
$\$_{ij}$ = unit cost of fuel at mine i plus transportation cost to plant j;
m_i = maximum output capacity of fuel at mine i;
r_i = thermal energy requirement of plant i;
k_i = calorific value of fuel from mine i;
s_i = percentage S in fuel from mine i [negative for virtual coal see (3b)];
a_i = percentage ash content in fuel from mine i (negative for virtual coal);
$(\bar{x}_i, \bar{y}_i)$ = location of mine i;
(x_i, y_i) = location of plant i.

The optimal plant sites must be determined subject to the following constraints:

(1) the thermal energy requirements of each plant must be met, i.e.

$$\sum_{i=1}^{M} k_i \chi_{ij} \geqslant r_j, j = 1, 2, \ldots N; \tag{16}$$

(2) the flow rate of fuel between any mine and any plant must not exceed the limiting rate of fuel transportation, i.e.

$$0 \leqslant \chi_{ij} \leqslant d_{ij}, \text{ all } i, j; \tag{17}$$

(3) the total rate of consumption of fuel from any mine cannot exceed the mines maximum output capacity, i.e.

$$\sum_{j=1}^{N} \chi_{ij} \leqslant m_i, i = 1, 2, \ldots M; \tag{18}$$

(4) plant sites must fall within the designated feasible area R, i.e.

$$(x_i, y_i) \varepsilon R, i = 1, 2, \ldots N. \tag{19}$$

Each of the above constraints can be expressed in the general form

$$G_l(\mathbf{z}) \leqslant 0, l = 1, \ldots J \tag{20}$$

where J is the total number of constraints.

Our search is for locations (x_j, y_j) of the N power plants that achieve optimal satisfaction of five separate objectives. These are

Objective (1): Minimization of fuel cost (fuel plus emissions control) for all plants combined. The combined fuel and control costs for plant j expressed as cost per unit time is

$$\$_j = \sum_{i=1}^{M} \$_{ij} \chi_{ij}. \tag{21}$$

Our objective is to minimize the total cost of all plants, namely minimize

$$F_1(\mathbf{z}) = \sum_{j=1}^{N} \$_j = \sum_{j=1}^{N} \sum_{i=1}^{M} \$_{ij} \chi_{ij}. \tag{22}$$

Here the cost factor $\$_{ij}$ can be expressed as

$$\$_{ij} = \$_i^f + \$_{ij}^t D_{ij} \tag{23}$$

where

$\$_i^f$ = cost per unit mass of fuel at mine i,
$\$_{ij}^t$ = cost of transportation of fuel per unit mass per unit distance along the line connecting mine i and plant j,
D_{ij} = transport distance which, for the purposes of this study, we will take to be
$$= [(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_j)^2]^{1/2}. \tag{24}$$
(The last two factors do not apply to virtual coal.)

Objective (2): Minimization of the probability of exceeding the short-term sulfur dioxide standard. Using expression (13) with $\tau = 24$ h and $C_M = 365\ \mu g\ m^{-3}$ we can express this objective in the form

$$\text{Minimize } F_2(\mathbf{z}) = C_S(\mathbf{z}_L, \mathbf{Q}_N) \tag{25}$$

where the components of Q_N are related to the components of the fuel allocation vector $\mathbf{z}_F$ through

$$Q_n = \sum_{i=1}^{M} \chi_{in} s_i. \tag{26}$$

Objective (3): Minimize the annual average SO_2 concentration in the airshed, i.e.

$$\text{Minimize } F_3(\mathbf{z}) = C_L(\mathbf{z}_L, \mathbf{Q}_N) \tag{27}$$

where $\mathbf{Q}_N$ and $\mathbf{z}_F$ are related by (26).

Objective (4): Minimize the probability of exceeding the 24-h particulate standard, namely

$$\text{Minimize } F_4(\mathbf{z}) = C_S(\mathbf{z}_L, \mathbf{Q}'_N) \tag{28}$$

where C_S is given by (13) with $\tau = 24$ h and $C_M = 260\ \mu g\ m^{-3}$, and the components of Q'_N are

$$Q'_n = \sum_{i=1}^{M} \chi_{in} a_i. \tag{29}$$

Objective 5: Minimize the annual average particulate concentration, i.e.

$$\text{Minimize } F_5(\mathbf{z}) = C_L(\mathbf{z}_L, \mathbf{Q}'_N) \tag{30}$$

where C_L is given by (14) and Q'_N and $\mathbf{z}_F$ are related through (29).

Our aim is to develop a procedure that will allow us to achieve optimal satisfaction of all five objectives (22, 25, 27, 28, 30) jointly while maintaining compliance with the system constraints (16, 17, 18, 19). This is termed a multi-criteria optimization problem, and we shall attempt now to solve it using concepts borrowed from game theory.

Game theory has been developed for both cooperative and non-cooperative games. In the latter, each

player acts independently in an effort to maximize his own gain. The outcome of such games is known as the Nash equilibrium solution to the nonzero-sum game. In the special case where only two players are involved and one's gain is the other's loss, the game is called a zero sum game and the associated Nash equilibrium solution reduces to the socalled saddle-point solution.

In cooperative games, players form coalitions under the expectation that working together each can achieve an outcome that is better than the Nash solution, i.e. the gain that each would have realized had they competed individually to maximize their gains. The measure of success of cooperative play is embodied in the concept of pareto-optimality which involves a single objective function defined by

$$F(\mathbf{z}) = \sum_{i=1}^{K} w_i F_i(\mathbf{z}) \tag{31a}$$

where F_i are objective functions associated with each of the K players and w_i is a weighting function with the properties

$$w_i \geq 0, \; i = 1, \ldots K \tag{31b}$$

$$\sum_{i=1}^{K} w_i = 1. \tag{31c}$$

Given a weighting function $w_i, i = 1, \ldots K$, a solution $\mathbf{z}$ that minimizes $F(\mathbf{z})$, as defined by (31a), is called a pareto-optimal solution, i.e. optimal in the sense that at least one player is better off and no player is worse off (see Zeleny, 1982, for further discussion of the concept of pareto-optimality). However, the solution that minimizes (31a) is clearly a function of the weighting function $\mathbf{w}$; and since some $\mathbf{w}$ give more weight to the objective functions of some players than others, implementation of the pareto-optimal objective requires that a function w_i, $i = 1, \ldots K$, be found that all players will agree yields a fair overall objective $F(\mathbf{z})$ of the game.

To find such a weighting function we begin by considering the situation where each of the K players is allowed in turn to find the design vector $\mathbf{z}$ that minimizes his objective function alone subject to the system constraints. Thus, we define the vector

$$\mathbf{z}_i^* = \text{value of } \mathbf{z} \text{ for which } F_i(\mathbf{z}) = \text{minimum}. \tag{31d}$$

By definition $F_i(\mathbf{z}_i^*)$ is a measure of the best outcome that player i could ever achieve in the game, but it is generally not an optimal outcome for any of the other players. That is,

$$F_j(\mathbf{z}_i^*) \neq \text{minimum for } i \neq j.$$

Using the $\mathbf{z}_i^*, i = 1, \ldots K$, and the objective functions $F_i(\mathbf{z}), i = 1, \ldots K$, evaluated at these points we can form the matrix

$$[\mu] = \begin{bmatrix} F_1(\mathbf{z}_1^*) & F_2(\mathbf{z}_1^*) & \ldots & F_K(\mathbf{z}_1^*) \\ F_1(\mathbf{z}_2^*) & F_2(\mathbf{z}_2^*) & \ldots & F_K(\mathbf{z}_2^*) \\ \vdots & \vdots & & \vdots \\ F_1(\mathbf{z}_K^*) & F_2(\mathbf{z}_K^*) & \ldots & F_K(\mathbf{z}_K^*) \end{bmatrix}. \tag{32}$$

This matrix contains useful information. In particular, the smallest element in column i is a measure of the best that player i can hope to achieve in the game under any conditions while the largest element in that column is the worst that player i could do in cooperative play. Consequently, we are led to define the 'super-criterion'

$$S(\mathbf{w}) = \prod_{i=1}^{K} \frac{\bar{F}_i}{F_i(\mathbf{z}_c)} \tag{33}$$

where $\bar{F}_i$ is the largest element in column i in $[\mu]$ and $\mathbf{z}_c$ is the design vector that minimizes

$$F_c(\mathbf{z}) = \sum_{i=1}^{K} w_i \frac{F_i(z)}{F_i^*} \tag{34}$$

where F_i^* is the smallest element in column i in $[\mu]$. Note that $S(\mathbf{w})$ is a measure of how far each player is from his *worst* outcome when the design vector is selected as the pareto-optimal solution based on the weighting function $\mathbf{w} = (w_1, \ldots w_K)$. In other words, in our search for a weighting function $\mathbf{w}$ that all players will agree best combines the individual objectives into a single objective F, given by (31), we are led to the function $\mathbf{w}$ that *maximizes* the super criterion (33). Because with that particular $\mathbf{w}$ player i is assured that in the specification of a single function F that best represents the objective of the coalition of players that he has joined, his individual objective F_i is given a weight w_i that will maximize his position in the game relative to the worst outcome he could suffer as a result of cooperative play.

Implementation of the pareto-optimal procedure is performed in the following steps:

Step 1. Construct the matrix $[\mu]$, (32), by solving K single criterion optimization problems.

Step 2. Choose any weighting function $\mathbf{w}$ that satisfies (31b, c) and use it in (34) to find the design vector $\mathbf{z}_c$ for which $F_c(\mathbf{z})$ is a minimum.

Step 3. Use the vector $\mathbf{z}_c$ obtained in Step 2 to evaluate the super criterion (33). Determine the change in $\mathbf{z}_c$ that would result in the largest increase in the super criterion.

Step 4. Iterate between Steps 2 and 3 until the super criterion has reached a maximum value.

4. ILLUSTRATION OF THE OPTIMIZATION PROCEDURES USING A HYPOTHETICAL PROBLEM

In this section we apply the techniques that we developed in the previous sections to a rather elementary problem involving the siting of $N = 2$ power plants of equal strengths Q in the region D containing $M = 2$ sources of coal. The objective is to find plant sites $\mathbf{X}_N = (x_{S1}, y_{S1}, x_{S2}, y_{S2})$ that minimize operating costs and air quality impacts jointly given information on the meteorology of the region, the cost and pollutant contents of the coal from each mine, etc. As we noted in the introduction, the principal aim of

this report is technique development rather than application, and therefore the problem that we will consider here is the simplest that will allow us to demonstrate key elements of the optimization procedure.

The factor that most strongly affects the utility of optimization techniques such as the present one is the form of the air pollution model, represented here in the concise forms (13) and (14). If the model is expressible only in terms of differential equations, which would generally be the case in treating photochemical species, the evaluation of objective functions F_2–F_5 might well require more computational effort than is feasible on present day computers. Therefore, we envisage the main use of optimization methods in problems concerning species such as SO_2 whose chemistry can be approximated by linear relationships, for which the kernels p_2 and p that enter in models (13) and (14) can be evaluated using meteorological data; and then expressed either as large matrices of discrete values, or piecewise polynomials fit to these values. We adopt the latter method for our sample problem, and implement it in the elementary form

$$C_s(\mathbf{X}_2, \mathbf{Q}_2) = Q \cdot [a_1 + a_2 x_{S1} + a_3 y_{S1} + a_4 x_{S2} + \ldots a_{19} y_{S2}^3] \quad (35)$$

with a similar expression for C_L.

In this sample problem we are looking for the vector $\mathbf{z}$ of design variables (see 15)

$$\mathbf{z} = \begin{bmatrix} x_{S1} \\ y_{S1} \\ x_{S2} \\ y_{S2} \\ \chi_{11} \\ \chi_{12} \\ \chi_{21} \\ \chi_{22} \end{bmatrix} \quad (36)$$

that results in optimal satisfaction of our cost and pollutant concentration minimization objectives while maintaining compliance with all system constraints. (In (36), χ_{ij} is the fuel flow rate between coal mine i and power plant j.) Following is a list of the assumed values for each of the system parameters:

	Coal mine 1	Coal mine 2
Coordinates	$(\bar{x}_1, \bar{y}_1) = (40, -20)$	$(\bar{x}_2, \bar{y}_2) = (50, -35)$
Fuel cost $\$^f$ [see (23)]:	$\$_1^f = \$0.40\ \text{kg}^{-1}$	$\$_2^f = \$0.60\ \text{kg}^{-1}$
Sulfur content of coal	$s_1 = 1\%$	$s_2 = 3\%$
Ash content of coal	$a_1 = 30\%$	$a_2 = 15\%$
Calorific value of coal	$k_1 = 4000\ \text{kcal kg}^{-1}$	$k_2 = 5000\ \text{kcal kg}^{-1}$
Maximum capacity of mine	$m_1 = 95\ \text{ton h}^{-1}$	$m_2 = 95\ \text{ton h}^{-1}$.

The assumed values of other system constraints include:

maximum fuel flow rate between mine i and power plant j $= d_{ij} = 45\ \text{t h}^{-1}$ (all i, j);

thermal energy requirement of plant i $r_1 = r_2 = 2.88 \times 10^8\ \text{kcal h}^{-1}$.

It is assumed that 99.5% of the fly ash is removed by control equipment at each plant. However, no control of S emissions is considered.

The first step in finding the optimal design vector $\mathbf{z}$ is to evaluate the matrix $[\mu]$, given by (32), whose elements are the five objective functions $F_i(\mathbf{z})$, $i = 1, \ldots 5$ [Equations (22), (25), (27), (28) and (30)], evaluated for the $\mathbf{z}_i^*$, $i = 1, \ldots 5$, which represent solutions of the extreme problems (31d). For example, $\mathbf{z}_1^*$ is the optimal design vector for the situation where objective function $F_1(\mathbf{z})$ is minimized without regard for any of the other four objectives. Tables 1–4 list the optimal vectors $\mathbf{z}_i^*$ and the associated value of the objective function, $F_i(\mathbf{z}_i^*)$, which were obtained for different initial guesses of $\mathbf{z}_i^*$. The point to note is that in optimization problems one is never assured of having found the global minimum of the objective function. One always finds local minima near the starting value of the design vector, so it is necessary to consider a number of widely scattered initial points to arrive at an estimate of the global optimum design. (For brevity we have omitted details on $\mathbf{z}_5^*$.)

Using the optimal vectors $\mathbf{z}_i^*$ found for each of the extreme problems, we form the matrix μ given by (32). We get

$$[\mu] = \begin{bmatrix} 119{,}690 & 70{,}931 & 862{,}059 & 2557 & 31{,}076 \\ 141{,}600 & 51{,}619 & 677{,}399 & 1888 & 24{,}782 \\ 139{,}291 & 61{,}683 & 382{,}997 & 2198 & 13{,}649 \\ 143{,}912 & 70{,}884 & 874{,}039 & 1318 & 16{,}247 \\ 152{,}780 & 82{,}455 & 522{,}618 & 1855 & 11{,}758 \end{bmatrix}.$$

Next we form the super criterion $S(\mathbf{w})$ defined by (33):

$$S(\mathbf{w}) = \frac{152{,}780}{F_1(\mathbf{z}_c^*)} \cdot \frac{82{,}455}{F_2(\mathbf{z}_c^*)} \cdot \frac{874{,}039}{F_3(\mathbf{z}_c^*)} \cdot \frac{2557}{F_4(\mathbf{z}_c^*)} \cdot \frac{31{,}076}{F_5(\mathbf{z}_c^*)}$$

Table 1. Optimum design vector $\mathbf{z}_1^*$ for the extreme problem in which the only objective is the minimization of cost

Initial design vector	Optimum design vector
$\mathbf{z}_1$ = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0); $F_1(\mathbf{z}_1)$ = 152,509	$\mathbf{z}_1^*$ = (42.6, 10.0, 80.0, 5.0, 44.4, 44.3, 22.1, 22.1); $F_1(\mathbf{z}_1^*)$ = 119, 731
$\mathbf{z}_1$ = (12.0, 22.0, 98.0, 22.0, 20.0, 40.0, 44.0, 30.0); $F_1(\mathbf{z}_1)$ = 162,686	$\mathbf{z}_1^*$ = (41.6, 10.2, 80.5, 5.0, 24.2, 33.2, 38.2, 31.1); $F_1(\mathbf{z}_1^*)$ = 124,823

The optimal vector $\mathbf{z}_1^*$ and the associated value of the objective function, $F_1(\mathbf{z}_1^*)$, are given for two different starting points $\mathbf{z}_1$ of the search procedure.

Table 2. Optimum design vector $\mathbf{z}_2^*$ for the extreme problem in which the only objective is minimizing short-term SO_2 concentration

Initial design vector	Optimum design vector
$\mathbf{z}_2$ = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0); $F_2(\mathbf{z}_2)$ = 76,636	$\mathbf{z}_2^*$ = (22.0, 16.5, 99.3, 14.8, 44.96, 45.0, 21.6, 21.6); $F_2(\mathbf{z}_2^*)$ = 51,674
$\mathbf{z}_2$ = (50.0, 20.0, 85.0, 22.0, 40.0, 44.0, 30.0, 24.0); $F_2(\mathbf{z}_2)$ = 75,354	$\mathbf{z}_2^*$ = (19.6, 16.8, 98.0, 14.5, 45.0, 45.0, 21.6, 21.6); $F_2(\mathbf{z}_2^*)$ = 51,771
$\mathbf{z}_2$ = (12.0, 12.0, 98.0, 24.0, 20.0, 25.0, 44.0, 40.0); $F_2(\mathbf{z}_2)$ = 78,611	$\mathbf{z}_2^*$ = (15.8, 17.2, 99.8, 15.0, 20.7, 25.6, 41.1, 38.0); $F_2(\mathbf{z}_2^*)$ = 67,343

The optimal vector $\mathbf{z}_2^*$ and the associated value of the objective function $F_2(\mathbf{z}_2^*)$ are given for three different starting points $\mathbf{z}_2$ of the search procedure.

Table 3. Optimum design vector $\mathbf{z}_3^*$ for the extreme problem in which the only objective is minimizing long-term SO_2 concentration

Initial design vector	Optimum design vector
$\mathbf{z}_3$ = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0); $F_3(\mathbf{z}_3)$ = 1120,455	$\mathbf{z}_3^*$ = (20.3, 10.1, 100.0, 7.8, 44.8, 44.9, 23.1, 22.9); $F_3(\mathbf{z}_3^*)$ = 383,186
$\mathbf{z}_3$ = (50.0, 20.0, 85.0, 22.0, 40.0, 44.0, 30.0, 24.0); $F_3(\mathbf{z}_3)$ = 1162,334	$\mathbf{z}_3^*$ = (49.4, 19.5, 83.6, 25.0, 41.6, 45.0, 24.7, 21.96); $F_3(\mathbf{z}_3^*)$ = 950,639
$\mathbf{z}_3$ = (12.0, 12.0, 98.0, 24.0, 20.0, 25.0, 44.0, 40.0); $F_3(\mathbf{z}_3)$ = 1156,838	$\mathbf{z}_3^*$ = (21.1, 10.2, 80.9, 25.0, 44.9, 44.5, 21.8, 22.1); $F_3(\mathbf{z}_3^*)$ = 769,642

The optimal vector $\mathbf{z}_3^*$ and the associated value of the objective function $F_3(\mathbf{z}_3^*)$ are given for three different initial points $\mathbf{z}_3$ of the search procedure.

Table 4. Optimum design vector $\mathbf{z}_4^*$ for the extreme problem in which the only objective is minimizing short-term particulate concentration

Initial design vector	Optimum design vector
$\mathbf{z}_4$ = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0); $F_4(\mathbf{z}_4)$ = 1839	$\mathbf{z}_4^*$ = (20.3, 15.4, 100.0, 15.9, 17.6, 17.6, 43.5, 43.7); $F_4(\mathbf{z}_4^*)$ = 1327
$\mathbf{z}_4$ = (32.5, 17.5, 90.0, 15.0, 40.0, 40.0, 30.0, 30.0); $F_4(\mathbf{z}_4)$ = 2023	$\mathbf{z}_4^*$ = (21.1, 18.2, 99.1, 13.5, 16.8, 16.1, 44.1, 44.8); $F_4(\mathbf{z}_4^*)$ = 1317
$\mathbf{z}_4$ = (50.0, 20.0, 85.0, 22.0, 40.0, 44.0, 30.0, 24.0); $F_4(\mathbf{z}_4)$ = 2448	$\mathbf{z}_4^*$ = (43.6, 25.0, 86.5, 18.5, 34.2, 40.1, 30.2, 25.5); $F_4(\mathbf{z}_4^*)$ = 1983

The optimal vector $\mathbf{z}_4^*$ and the associated value of the objective function $F_4(\mathbf{z}_4^*)$ are given for three different initial points $\mathbf{z}_4$ of the search procedure.

in which $\mathbf{z}_c^*$ is the vector that minimizes the combined objective

$$F_c(\mathbf{z}) = w_1 \frac{F_1(z)}{119{,}690} + w_2 \frac{F_2(z)}{51{,}619} + w_3 \frac{F_3(z)}{382{,}997} + w_4 \frac{F_4(z)}{1318} + w_5 \frac{F_5(z)}{11{,}758}$$

subject to constraints (16–19). Following the optimal search procedure outlined earlier, we arrive finally at the pareto-optimal design $\mathbf{z}_c^*$ given in Table 5.

Recall that the elements of the matrix $[\mu]$ given above are the objective function values $F_i(\mathbf{z}_i^*)$, which represent the degree to which objective i is satisfied in the situation where an attempt was made to meet only

Table 5. Pareto-optimal design vector $\mathbf{z}_c^*$ obtained using game theory concepts

Initial values	Final, optimal values
$\mathbf{w} = 0.1312:0.1626:0.2357:0.1997:0.2707$	$\mathbf{w}^* = 0.146:0.1945:0.193:0.2659:0.2095$
$\mathbf{z}_c = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0)$	$\mathbf{z}_c = (32.5, 17.5, 90.0, 15.0, 30.0, 30.0, 40.0, 40.0)$
$\mathbf{z}_c^* = (29.2, 12.6, 98.6, 8.0, 38.4, 38.3, 39.8, 39.8)$	$\mathbf{z}_c^* = (24.4, 11.6, 100.0, 7.5, 34.8, 34.7, 35.0, 35.0)$
$S(\mathbf{w}) = 2.66$	$S(\mathbf{w}) = 5.98$ with $F_1(\mathbf{z}_c^*) = 150{,}713$ $F_2(\mathbf{z}_c^*) = 75{,}020$ $F_3(\mathbf{z}_c^*) = 487{,}562$ $F_4(\mathbf{z}_c^*) = 2019$ $F_5(\mathbf{z}_c^*) = 13{,}124$

objective j. Comparing those 'non-cooperative game' results with the objective function values given in Table 5 for the outcome of the cooperative optimization effort, we see that the effort to satisfy the five objectives jointly has resulted in design values that satisfy each objective to a degree intermediate between the worst and best that would result if the objectives were treated individually. It appears that using the starting vector $\mathbf{w}$ given in Table 5, the pareto-optimal outcome favors objectives 4 and 5 more than 1 and 2 inasmuch as objective functions $F_4(\mathbf{z}_c^*)$ and $F_5(\mathbf{z}_c^*)$ given in the table are closer to their absolute minimum values, given by the corresponding diagonal elements of $[\mu]$ above, than the pareto-optimal values of F_1 and F_2. Finding the global optimum vector $\mathbf{z}^*$ requires that the process summarized in Table 5 be repeated for different initial values of the weighting vector $\mathbf{w}$.

5. CONCLUSION

We have developed a procedure based on concepts of game theory by which one can utilize air pollution models to find optimal sites for industries that emit air pollutants. The technique enables one to find plant sites that result in optimal compliance with multiple objectives subject to given constraints. The objectives we considered are minimization of cost, including the cost of operating pollution control equipment; and the minimization of the peak and long-term averaged concentrations of each of two air pollutants—a total of five objectives in all. One of the useful functions that a technique of this type can provide is the ability to assess the penalties associated with efforts to satisfy one or several objectives at the exclusion of others. For example, how much increase in long-term averaged concentration would result if only operating cost and peak concentration are minimized jointly? Or, what sacrifice in the attainment of a given secondary standard would result if the only additional emissions controls added were those that effected minimum cost and minimal violation of the standard jointly? These are questions of potentially far reaching significance which we hope to pursue in a future study.

Although the technique that we have developed has its merits, it also has drawbacks. One of these is a problem common to all optimization procedures, namely, the inability to determine that a given solution is the global optimum. The method produces solutions that are optimum within some local region of the parameter space; but to find the global minimum solution, one must generally apply the method to a number of different starting points in the parameter space and select from the resulting set of local optimum solutions the one that best satisfies the objective function. This procedure can be expensive computationally, especially if the air quality objectives are expressed in terms of a model or models consisting of differential equations. For this reason, we believe that the only feasible application of our technique, with present generation computers, is in problems where the air quality objectives are expressible either in closed functional forms such as (35), or as matrices of discrete values.

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